occur. The importance of measuring both the drift mobility and the Ohmic conductivity as a function of temperature in order to unambiguously establish the presence of a hopping mode of conduction has been clearly demonstrated by the work of LeComber and Spear.[1]

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Temperature Dependence of Electron Mean Free Paths in Cadmium and Copper*

W. Royall Cox and J. D. Gavenda
Department of Physics, The University of Texas, Austin, Texas 78712
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A T^{-5} temperature dependence of electron mean free paths is deduced from high-field and open-orbit ultrasonic attenuation data in cadmium and copper.

I. INTRODUCTION

The possibility of a difference between the electron mean free paths for ultrasonic attenuation and electrical resistivity has been discussed by Steinberg[1] and by Bhatia and Moore.[2] Subsequently, low-temperature ultrasonic attenuation experiments in potassium[3] and copper[4] (in the absence of a magnetic field) have yielded mean-free-path values which vary with temperature as T^{-5}, in agreement with the Bloch-Grüneisen theory.[5] In other metals, however, deviations from this theory have been found in both attenuation and resistivity data, and the two experimental techniques sometimes appear to give different results.[6–9] For example, an analysis by Deaton[6] of high-field and open-orbit-resonance data in cadmium indicated temperature dependences for the mean free paths which vary between T^{-3} and T^{-4} (above 3 °K), although the resistivity data appear[10] to follow a T^{-5} law. These results suggested the possibility of a magnetic field effect on the mean free paths,[6] and this question was brought to mind more recently by the $T^{-3.6}$ temperature dependence found for mean free paths in thalluim[8] by magnetoacoustic experiments. We wish to show here that a somewhat more rigorous analysis of Deaton's cadmium data indicates a variation of the mean free paths as T^{-5}, in agreement with the resistivity measurements, and that the same result is obtained when the analysis is applied to similar magnetoacoustic data in copper.

II. THEORY

Our analysis of the low-temperature ultrasonic attenuation data begins with the usual assumption that the mean free path l can be expressed according to Matthiessen's rule,[5]

$$l^{-1} = l_0^{-1} + l_{ph}^{-1}, \qquad (1)$$

where l_0 and l_{ph} are the mean free paths for impurity and phonon scattering, respectively. We further assume that at very low temperatures ($T \ll \Theta$)

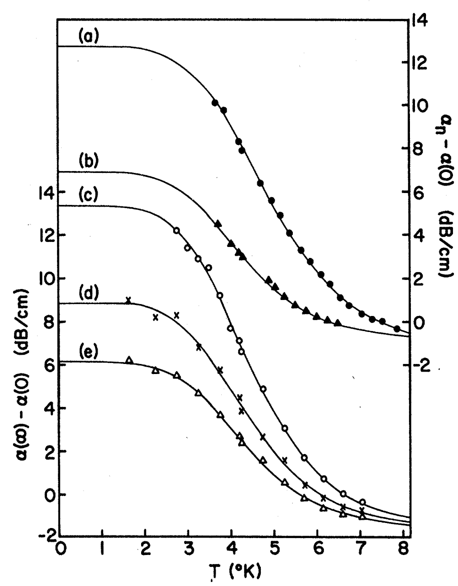

FIG. 1. Least-squares computer fit of ultrasonic attenuation data in cadmium as a function of temperature. The relative open-orbit-resonance height (right-hand scale) is for (a) $f = 72$ MHz with $\vec{q} \parallel [10\bar{1}0]$ and $\vec{B} \parallel [\bar{1}2\bar{1}0]$ and for (b) $f = 31$ MHz with $\vec{q} \parallel [\bar{1}2\bar{1}0]$ and $\vec{B} \parallel [10\bar{1}0]$. The relative high-field attenuation (left-hand scale) is for $f = 31$ MHz and $\vec{q} \parallel [\bar{1}2\bar{1}0]$ with (c) $\vec{B} \parallel [10\bar{1}0]$ and with $\vec{B}$ rotated, (d) 5°, and (e) 10° from $[10\bar{1}0]$ in the $(\bar{1}2\bar{1}0)$ plane.

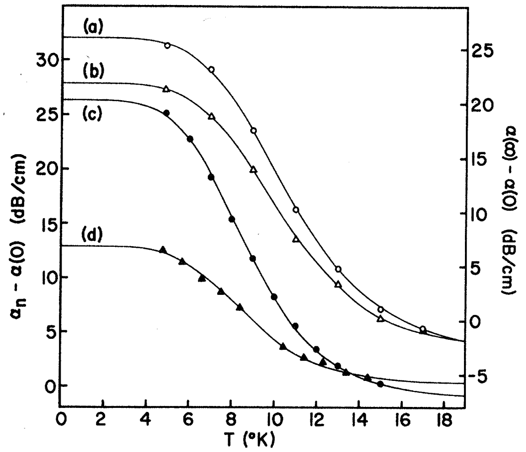

FIG. 2. Least-squares computer fit of ultrasonic attenuation data in copper as a function of temperature. The relative high-field attenuation (right-hand scale) is for $f = 33$ MHz and $\vec{q} \parallel [\bar{1}01]$, with (a) $\vec{B} \parallel [111]$ and (b) $\vec{B} \parallel [1\bar{2}1]$. The relative open-orbit-resonance height (left-hand scale) is for (c) $f = 53$ MHz and (d) $f = 32$ MHz, with $\vec{q} \parallel [\bar{1}01]$ and $\vec{B} \parallel [1\bar{2}1]$.

$$l_{\text{ph}}^{-1} \propto (T/\Theta)^N, \quad (2)$$

where N is a constant (ideally, $N = 5$ for the electrical resistivity case) and Θ is the Debye temperature. Then the resultant mean free path may be written in the form

$$l = \frac{l_0}{1 + C(T/\Theta)^N}, \quad (3)$$

where the value of C for a given metal is proportional to l_0. Accordingly, any quantity which varies directly with l, such as the height α_n of an open-orbit peak[11] or the level $\alpha(\infty)$ at which the compressional-wave attenuation saturates in large magnetic fields,[12,13] should exhibit a temperature dependence of the form of Eq. (3). Since the attenuation $\alpha(0)$ in the absence of a magnetic field is independent of l as long as the mean free path is much longer than a wavelength of sound,[12–14] i.e., when $ql \gg 1$, we would expect in this limit to be able to fit *relative* open-orbit and high-field data $\alpha_n - \alpha(0)$ and $\alpha(\infty) - \alpha(0)$, respectively, to a function of the form

$$\alpha_{\text{rel}} = \frac{A}{1 + C(T/\Theta)^N} - B. \quad (4)$$

Here B represents $\alpha(0)$ which we treat as a free parameter because the experiments give only the *change* in attenuation caused by the magnetic field. In addition to the electronic attenuation other mechanisms produce a background attenuation, which we assume to be field independent.

III. RESULTS AND CONCLUSIONS

All of the data reported here were obtained using a standard pulse-echo technique[15] to measure the relative attenuation, and a helium-bath and heater arrangement[6] to vary the temperature. A least-squares computer fit to Eq. (4) with $N = 5$ is shown in Fig. 1 for some of the cadmium data previously presented by Deaton,[6] and in Fig. 2 for similar data from a single crystal of copper[16] having a residual resistivity ratio of 35 000. For consistency, values of Θ deduced from low-temperature heat-capacity measurements (209 and 343 °K for cadmium and copper, respectively,[17] were used for fitting both sets of data. Since $ql \gtrsim 7$ when $\alpha(\infty) - \alpha(0) \gtrsim 0$,[18] we see that the condition $ql \gg 1$ is satisfied at temperatures below about 6 and 15 °K for cadmium and copper, respectively, so the use of Eq. (4) is justified in this case. [At higher temperatures Eq. (4) may be approximated by the fitting function used by Deaton[6]: $\alpha_{\text{rel}} = C_1 + C_2 T^{-N}$.]

One would expect the constants A and B of Eq.

TABLE I. Constants determined by fitting the attenuation data of Figs. 1 and 2 to a function of the form of Eq. (4) with $N = 5$ and with the Debye temperature Θ taken (after Ref. 17) to be 209 and 343 °K for cadmium and copper, respectively.

Metal	Dependent variable	$\vec{q}$	Frequency (MHz)	$\vec{B}$	A (dB/cm)	B (dB/cm)	$10^{-7}C$	DEV (dB/cm)
	$\alpha_n - \alpha(0)$	$[10\bar{1}0]$	72	$[\bar{1}2\bar{1}0]$	14.41	1.67	13.66	0.10
Cd		$[\bar{1}2\bar{1}0]$	31	$[10\bar{1}0]$	7.94	1.01	26.79	0.07
	$\alpha(\infty) - \alpha(0)$	$[\bar{1}2\bar{1}0]$	31	$[10\bar{1}0]$	15.27	1.88	21.40	0.19
				$[10\bar{1}0]$ + 5°	10.66	1.78	23.15	0.19
				$[10\bar{1}0]$ + 10°	7.92	1.79	23.82	0.09
	$\alpha_n - \alpha(0)$	$[\bar{1}01]$	53	$[1\bar{2}1]$	27.78	1.49	9.07	0.19
Cu			32	$[1\bar{2}1]$	12.93	0.04	9.62	0.20
	$\alpha(\infty) - \alpha(0)$	$[\bar{1}01]$	33	$[1\bar{2}1]$	24.88	3.04	2.84	0.16
				$[111]$	29.50	3.47	3.31	0.13

(4) to depend upon such properties of the metal as density, electron concentration, and Fermi velocity and to vary, in the free-electron approximation, as l_0q^2 and q, respectively.[11–13] (Here $q=\omega/v_s$, where ω and v_s are the sound frequency and velocity, respectively.) Also, according to the Bloch-Grüneisen formalism,[5] C should depend upon the Fermi velocity v_F and should vary approximately as Θ^{-1}. Thus, we might expect C to be somewhat larger for cadmium than for copper (if the values of l_0 and v_F are roughly equivalent for the two metals) because of the difference in Debye temperatures.

The values of the constants of Eq. (4) determined for each set of experimental points with $N=5$ are listed in Table I, together with the root-mean-square deviations (DEV) indicating the quality of the fits. It can be seen that A and B scale roughly with frequency and that C is larger for cadmium than for copper. However C appears to differ somewhat between the open-orbit and high-field data, and all of the constants are quite sensitive to the sound propagation direction $\vec{q}$ and the magnetic-field orientation $\vec{B}$. This is basically a reflection of the widely varying contributions to the attenuation from different parts of the Fermi surfaces of these metals and can be attributed to orbit geometry (especially the orientation of open orbits relative to $\vec{q}$) and to mean-free-path and deformation-potential anisotropy.[13,19]

The inseparability[19] of these effects precludes any reliable estimates of l_0 from Table I, but the high quality of the fits (as reflected by the relatively small values of DEV) indicates that the average temperature responses of the mean free paths in these metals can be described by a T^{-5} law. When the uniqueness of the power law for these data was checked by allowing N to vary in Eq. (4), it was found in each case that N differed from 5 by no more than a few percent and that very little improvement in DEV could be obtained by using a four-constant fit. The validity of the assumption that the values of N deduced from the fitting process give the correct temperature dependences of the mean free paths was confirmed by the fact that values of l extracted from the relative widths of the open-orbit peaks in copper exhibit approximately the same temperature variation as the relative peak amplitudes $\alpha_n - \alpha(0)$.

Since all of these data indicate a T^{-5} temperature behavior for the mean free paths, it can be concluded that the presence of a magnetic field does not affect the temperature sensitivity of l and that in cadmium and copper there is no significant difference in the temperature dependences of the mean free paths for ultrasonic attenuation and electrical resistivity.

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